

Implementation and Validation of a Co-rotated Shell Element in the FEM-Software Carat++

A non-linear triangular shell element based on the corotational formulation proposed by B. Haugen in his PhD thesis [1] is implemented in the FEM software CARAT++. In this master thesis, the whole formulation is studied including its incorporation with the 3D finite rotation theory and an initial stress extension is provided. The robustness of the implementation and the behavior of the co-rotated element is tested by applying well-known non-linear benchmarks.

Comparison of the Corotational Formulation with the Total Lagrangian and the Updated Lagrangian Kinematic Descriptions

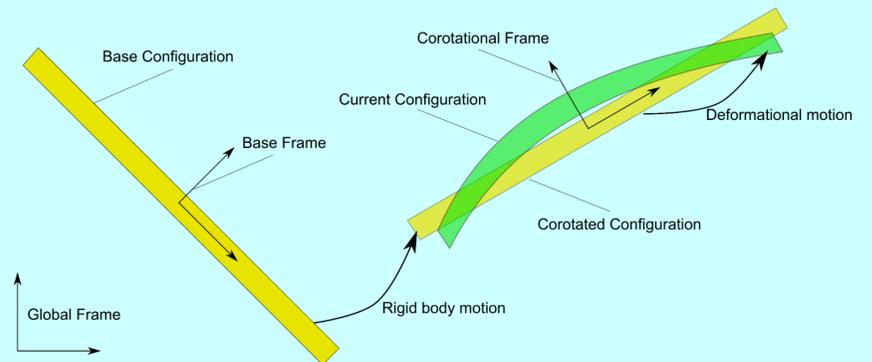
Advantages:

- Allows very large rigid body motions (Rotations and Translations)
- Element independent (Linear part of the stiffness matrix can be exchanged with another linear element if the geometry is the same)

Restriction:

- Although the displacements and rotations can be large, the deformations must remain small.

Kinematics of the CR Formulation

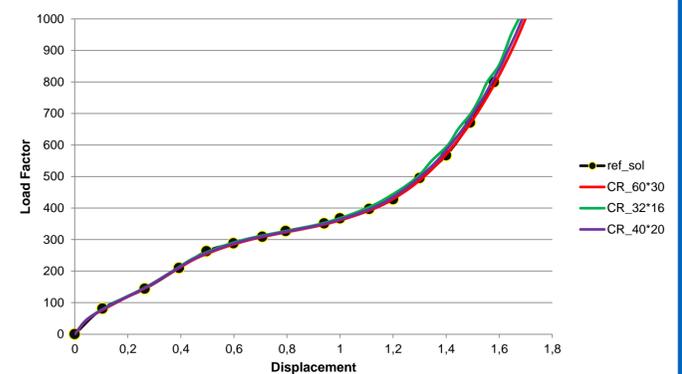
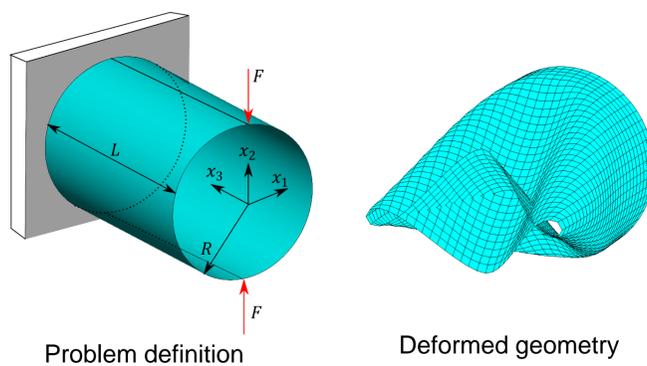


Benchmarks

The implemented non-linear shell element is tested with several well known benchmarks.

Pinching of a clamped cylinder: One end of the cylinder is clamped and at the other end two point loads are acting on the structure as depicted in the problem definition. For this benchmark, different mesh densities are used and the main purpose of applying this benchmark is to test the robustness of the formulation and the implementation.

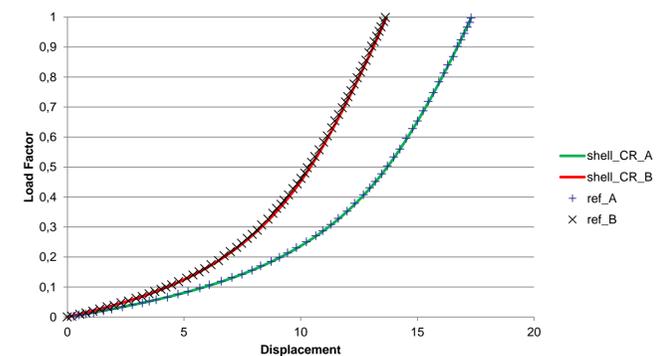
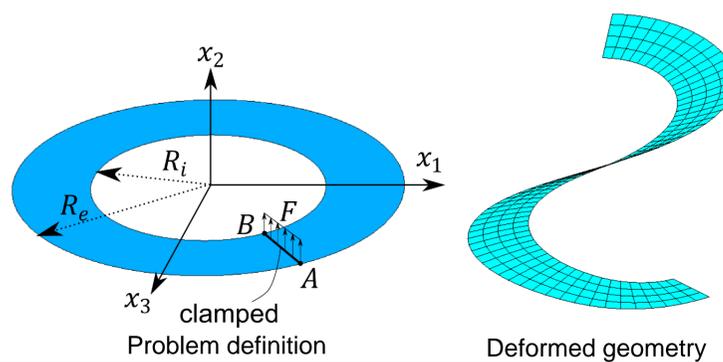
Pinching of a clamped cylinder



Two of the applied benchmarks are able to show the problem types where the CR formulation can be dominantly used.

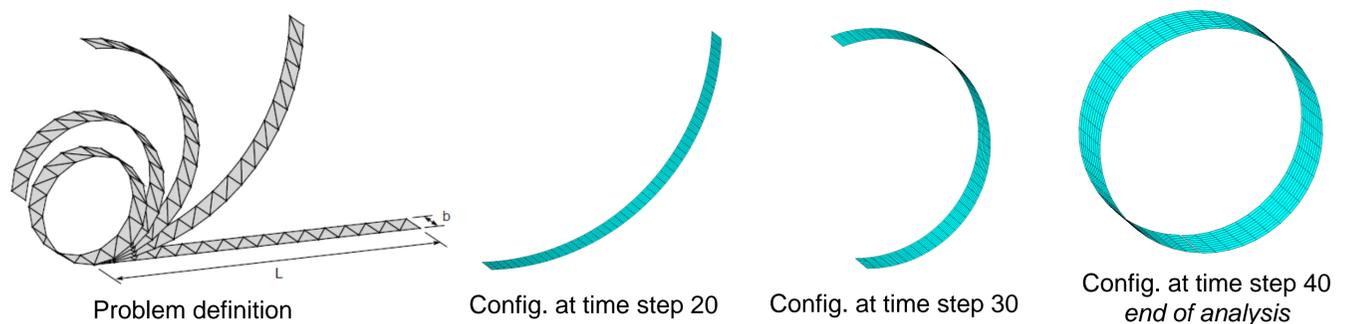
Slit annular plate subjected to lifting line force: Involves very large rigid body translations. The vertical displacement of the points A, B are much larger than the outer Radius ($R=10$); This benchmark proves that the CR formulation can handle very large rigid body translations.

Slit annular plate subjected to lifting line force



Cantilever beam subjected to end moment: According to the Bernoulli Euler theory, there is a relation btw. the end moment and the radius of a clamped beam which is $M_y = EI/R$. With the proper moment and geometry, at the end of the analysis a perfect circle is obtained. This benchmark proves that the CR formulation can handle very large rigid body rotations.

Cantilever beam subjected to end moment

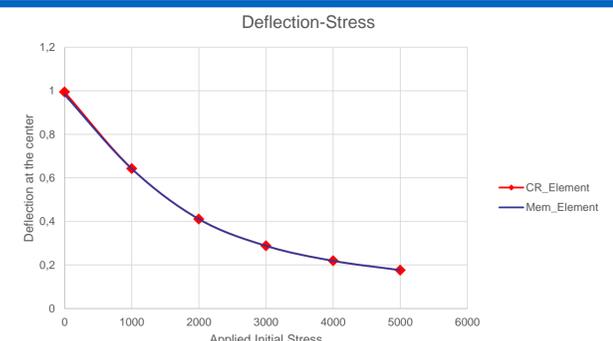


Initial Stress

The CR formulation proposed by Haugen is not derived with the initial stress assumption. In this thesis an initial stress extension is provided to the CR element by using the theory and implementation of a membrane element which is formulated with the initial stress assumption [2].

Initial Stress Benchmark Definition;

A dead load is applied on a square plate at the beginning of the analysis and the initial stresses in the x and y direction are increased gradually. Consequently, the deflection at the center of the plate is decreased. The CR element is compared with the membrane element [2], and the thickness is specified so small that CR element shows membrane properties.



References;

- [1] B. Haugen, Buckling and stability problems for thin Shell structures using high-performance finite elements, Ph.D. Dissertation, Department of Aerospace Engineering Sciences, University of Colorado, Boulder, CO, 1994
- [2] F. Dieringer, Implementierung eines geometrisch nichtlinearen Membranelements in einer objektorientierten Programmierumgebung, Master Thesis, Faculty of Civil Engineering, Technische Universität München, March 2009